A Graph-based Heuristic for Variable Selection in Mixed Integer Linear Programming

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Over the past decades, Branch and Bound (B&B) and its variants have become the most commonly used algorithms to solve Mixed Integer Linear Programming (MILP). Despite the efficiency of such methods, state-of-the-art solvers still rely on generic heuristics due to a lack of mathematical understanding of the underlying mechanics of the B&B algorithm. Amongst all the decisions made heuristically in a B&B tree, the most elementary are variable selection (branching) and node selection. As pointed out in [1], branching is crucial in controlling the size of a B&B tree and a plethora of heuristics has been put forward in the literature.

In this work, we propose a new branching heuristic, based on a graph representation of a MILP. Unlike many LP-based branching heuristics (e.g. Strong Branching and its variants), we propose an Influence Maximization scheme to target influential variables in the system of constraints. Indeed, the modifications of the polyhedron induced by branching in a B&B tree can be seen as local and guided by the links between variables through the set of constraints. This led us to the definition of a class of Influence Graphs for MILP dedicated to the proposed heuristic. As Influence Maximization in graphs is an NP-hard problem [2], we opt for using an approximation based on Spectral Clustering, allowing us to target multiple candidates for branching. We show that the proposed heuristic is equivalent in a specific case to clustering variables in a low-dimensional space when considering a MILP problem as a scatter of points.

We run some experiments with various Influence Graphs on EDF's energy planning problems, showing promising results in terms of size of the generated tree. The proposed heuristic seems to make structural branching decisions when used close to root node. However, the efficiency of this approach depends on the formulation as the notion of Influence is not invariant to reformulation.

References

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